MPC for Disturbance Rejection in a Quadcopter

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***Abstract*— Extensive work has focused on controlling quadcopter flight patterns in controlled environments, but little has been studied in natural environments. Building from works by Masse et al. and Alexis et al., this work proposes a hierarchical control structure for a quadcopter operating in windy conditions with a goal of waypoint tracking. We present an MPC design using LQR to ensure recursive feasibility and stability, and simulate wind disturbances modeled as a constant wind, a gust of wind, and erratic wind. Our controller demonstrates the ability to reject disturbances while maintaining stability throughout flight. Continuation of this work on physical systems would substantiate these simulation results and inform better control designs for aerial vehicles. A video presentation of this report is available to view at: `https://youtu.be/TyYSpawPRvc`.**

I. Introduction

Modern technology and the commercialization of drones have made quadcopters commonplace in our society. However, significant research remains to be done to improve their mobility and to make them agile and robust in natural environments. Because the medium through which quadcopters locomote is the lower atmosphere, they are subject to numerous disturbances in their flight that are impossible to predict with onboard controllers. Disturbances such as impacts with flying animals, fixed obstacles, wind, and rain present particular challenges. In this paper, we choose to focus our efforts on how wind disturbances may affect quadcopter flight, and the ways in which a high-level model predictive controller could adapt to these perturbations robustly.

A. Background

Significant literature exists regarding intelligent control of quadrotors in a variety of environments. Model predictive control (MPC) has been implemented onboard quadcopters on several occasions [1], [2]. However, as stated in Bangura et. al, “the underlying computational costs of full nonlinear MPC control is prohibitive.” As a result, most works which focus on MPC for real-time deployment rely on highly linearized models in which only a higher-level controller is regulated, and we take a similar approach in this work.

Because many companies in recent years have begun investigating potential drone usage for delivering goods across a particular geographic area efficiently, there exists a large body of work devoted to studying how disturbances from an external payload affect quadrotor flight. In particular, some recent works have developed robust controllers for adapting to off-center payload mass [3] or payloads on flexible tethers [4], multi-quadrotor systems aiming to collectively carry a larger payload [5], [6], and how such controllers might adapt to a rapidly varying payload center of mass location [7].

However, significantly less research exists regarding quadcopter control design for robustness to natural disturbances like wind, but some primary examples do exist in the very recent literature. For example, Allison et al. utilize Proportional-Integral-Derivative (PID) control to respond to wind disturbances during waypoint and trajectory tracking for a quadcopter [8], while PID and Linear Quadratic Regulator (LQR) control are implemented to control a hovering quadcopter subject to disturbances [9]. Similarly, Bannwarth et al. examine the deployment of a disturbance accommodating controller (DAC) and a nonlinear feedforward controller to regulate quadcopter flight [10]. Lastly, Masse et al. evaluate the performance of a quadcopter subject to increasing wind loads under both LQR control and structured $\mathscr{H}_\infty$ synthesis [11].

B. Contribution

This report presents one of the few analyses of MPC implementation for real-time quadrotor control, subject to natural disturbances such as wind. Due to the challenges of implementing MPC with a computation speed sufficient for real-time control, this work and most works on the topic primarily use simulation to inform control design and/or use highly linearized models. Future work would bring our preliminary simulation analysis into a broader investigation of MPC for wind rejection on a physical system for comparison to results from less advanced control techniques.

C. Overview

In this paper, we first introduce dynamical equations that have traditionally been used to model quadrotors in Section II-A, and the linearized approximate equations of motions are provided in Equation 1. The overarching control heirarchy for quadcopters based upon these dynamical equations is presented in Section II-B, along with assumptions made in the formulation of our control system. Section II-C then discusses the modeling of disturbances to the quadcopter during flight, and the resulting integration of these disturbances into our system dynamics.

In Section III-A, we introduce a Constrained Finite Time Optimal Control (CFTOC) problem, and its associated constraints and cost functions. Section III-B then discusses the implementation of this finite time control problem into a model-predictive control algorithm which tracks some terminal reference state. Lastly, the MPC is modified in Section III-C to accommodate flight disturbances such as steady

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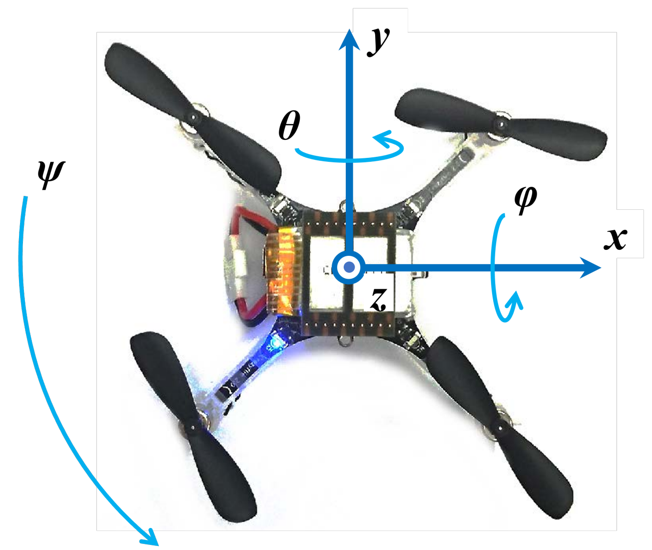

Fig. 1. **Parameters defining quadcopter orientation** Angles ϕ, θ, and ψ represent roll, pitch, and yaw, respectively, of the quadcopter in the inertial frame. The variables x, y, and z are chosen to represent position of the quadcopter center of mass in the inertial frame. Adapted from [12].

state wind, short impulses such as wind gusts, and random noise from inconsistent wind. Discussion of the controller's performance and areas for future work on this problem are presented in Section IV.

II. Modeling

A. System Dynamics

We model the states of our quadcopter according to the axes illustrated Figure 1.

The linearized and approximate equations of motion for the quadcopter are listed in Equation 1 and derived in [12].

$$\dot{\mathbf{x}} = \frac{d}{dt}\begin{bmatrix} x \\ y \\ z \\ \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ ||g||\theta \\ -||g||\phi \\ \frac{\Delta c_\Sigma}{m} \end{bmatrix} \quad (1)$$

$\mathbf{x}$, in bold, is shorthand for the state vector of position and velocity. $||g||$ is the magnitude of gravity, Δc_Σ is the total force from the propellers minus the effect of gravity, and m is the mass of the quadcopter.

The optimization problem and physics simulations presented in this report discretize the continuous dynamics of Equation 1 by the forward Euler method.

B. Control Hierarchy and Assumptions

The angle ψ has no effect on our states of interest. θ and ϕ are modeled as inputs. The result is a control system architecture, as illustrated in Figure 2. $\mathbf{x}_{ref}$ is the reference state and $\mathbf{u}$ represents the optimal inputs to the quadcopter: ϕ, θ, and Δc_Σ, which are generated by the CFTOC problem, and $\mathbf{x}_{reallifeattimet}$ represents the true state after the plant is

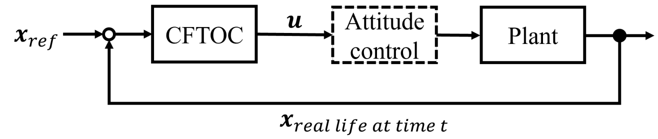

Fig. 2. **Quadcopter control architecture** The various segments of our chosen MPC controller, beginning with a CFTOC which has knowledge of the reference and actual states. The optimal inputs calculated by the CFTOC are then fed into an ideal lower level attitude controller which controls motor thrusts of the plant, the quadcopter, in its environment.

subjected to $\mathbf{u}$ and forces not included in the model, such as wind disturbance. The attitude control block is included in dashed lines to show that, in practice, a lower level controller would manipulate propeller forces to follow ϕ, θ, and Δc_Σ signals from the CFTOC. However, in this report we assume that this lower level controller can track these angles with a high enough bandwidth that the angular dynamics can be neglected, as is assumed in [13].

Another important assumption made for the sake of this report is that our knowledge of the quadcopter state is perfect. In practice, of course, the accuracy of this knowledge depends on the vehicle's sensors and state estimator.

C. Modeling of Flight Disturbances

Wind has been characterized in previous literature as the Dryden Wind model, which consists of a constant directional component plus some time-varying component [8], [9], [11]. Other works utilize a variety of modeling assumptions for wind such as white gaussian noise [11]. In this report, we study three types of wind: steady wind, a gust of wind, and erratic winds, which are modeled as a constant signal, a square wave, and a Gaussian noise centered around zero, respectively. Each is applied individually as a disturbance to the quadcopter, and these disturbances are not modeled in the CFTOC.

III. Controller Design

Utilizing the discretized system dynamics with a timestep of $Ts = 1/10$ seconds, outlined in Section II-A and denoted as $\mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k$, we aim to control the quadcopter's flight between two stationary points, when subjected to various forms of disturbance, described in Section II-C. We define a CFTOC problem with the introduction of a set of system constraints to ensure safe translational and rotational speeds as well as reasonable thruster forces, denoted as $\mathbf{x}_k \in \mathscr{X}$ and $\mathbf{u}_k \in \mathscr{U}$ for $k = 0, 1, \ldots, N-1$, and $|\mathbf{u}_{k+1} - \mathbf{u}_k| \leq u_d$. The system constraints were selected as follows:

- $|x, y, z| \leq 1000m$
- $|\dot{x}, \dot{y}, \dot{z}| \leq 2m/s$
- $|\theta, \phi| \leq 30°$
- $|\Delta c_\Sigma| \leq 50N$
- $u_d = 1°/s$ or $1N/s$

A. Finite Time Optimal Control

Employing LQR, we aim to minimize a quadratic cost function while tracking a terminal reference state. Initially,

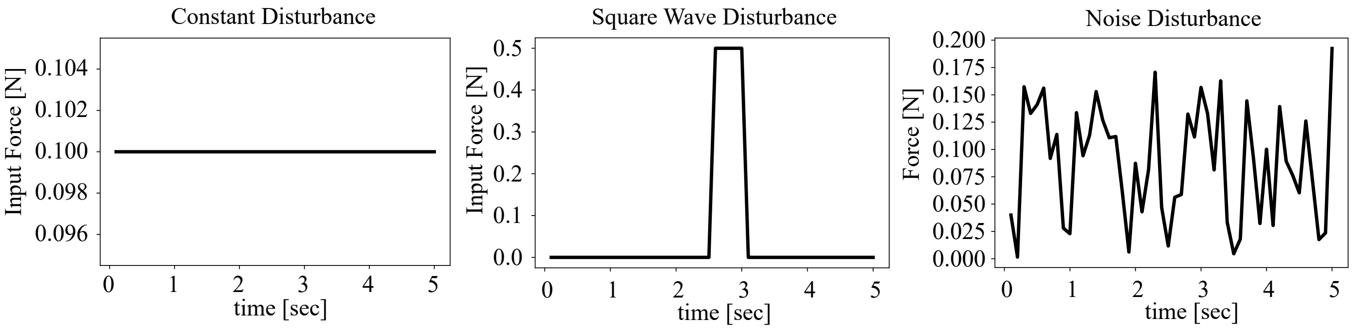

Fig. 3. **Choices for Wind Disturbances** Constant, square wave, and noise disturbance inputs (in Newtons) fed into the high-level MPC controller.

we set the terminal cost (P) and input cost (R) matrices to identities and introduce a large penalty on all of the states through Q, depicted in the CFTOC problem formulation below, with the previously mentioned timestep and a horizon time of $N = 50$, equivalent to a total duration of 5 seconds:

$$
\begin{aligned}
\min_{x,u} \quad & (\mathbf{x}_N - \bar{\mathbf{x}}_{ref})' P (\mathbf{x}_N - \bar{\mathbf{x}}_{ref}) + \sum_{k=0}^{N-1} ((\mathbf{x}_k - \bar{\mathbf{x}}_{ref})' Q (\mathbf{x}_k - \bar{\mathbf{x}}_{ref}) \\
& + \mathbf{u}_k' R \mathbf{u}_k) \\
\text{s.t.} \quad & \mathbf{x}_{k+1} = A\mathbf{x}_k + B\mathbf{u}_k, \forall k = 0, ..., N-1 \\
& \mathbf{x}_0 = \mathbf{x}(0) \\
& \mathbf{x}_N \in \mathscr{X}_f \\
& \mathbf{x}_k \in \mathscr{X}, \forall k = 0, ..., N-1 \\
& \mathbf{u}_k \in \mathscr{U}, \forall k = 0, ..., N-1 \\
& |\mathbf{u}_{k+1} - \mathbf{u}_k| \leq u_d
\end{aligned} \tag{2}
$$

An initial test of flying an undisturbed quadcopter from the origin to position $(1, 1, 1)$ at this horizon length demonstrated the feasibility of the CFTOC problem in 5 seconds, thus we use the same CFTOC problem in the implementation of a MPC.

B. MPC Implementation

We first identify a new terminal cost matrix and continuous Lyapunov function, P_∞, by solving the discrete time Algebraic Riccati Equation and implement a simple stabilizing controller with LQR gain K. With such a stabilizing controller, we design an MPC that can fly the quadcopter from the starting position to the maximal invariant set (O_∞) of the constrained closed loop state feedback system by setting the terminal set, $\mathscr{X}_f = O_\infty$, thus guaranteeing terminal set invariance. Since we previously tuned our state and input cost matrices to be positive definite, we ensure a persistently feasible MPC control law with these three assumptions [14]. We further show that using Equation 2 with a horizon length of $N = 5$ simulated M = 50 times with the same discretized timestep, the quadcopter can successfully reach O_∞ in 5 seconds and generate an optimal input sequence.

C. MPC Response to Disturbances

To evaluate the robustness of the designed controller, we introduce three types of disturbance forces to the system. As shown in Figure 4b, a steady wind disturbance modeled as a constant signal with a gain of 0.1N yields a large control input immediately, which reduces to a relatively constant control input after approximately one second. The angular inputs, θ and ϕ, each reach a magnitude of approximately 30 degrees, before settling to around 5 degrees. The thrust force minus the force of gravity, Δc_Σ, immediately spikes twice to counteract the wind, before settling to about $-0.5N$.

When a gust of wind is applied to the system and modeled as a square wave between 2.5-3 seconds with a gain of 0.5N, the quadcopter reaches the final position and successfully rejects the square wave wind disturbance, as depicted in Figure 4a. The control inputs resemble those for the constant disturbance, with a second region of spiking in both angular and force inputs that last the duration of the disturbance.

Lastly, with erratic winds modeled by Gaussian noise centered at 0 throughout the duration of the simulation, the quadcopter reaches the terminal destination in a similar fashion as with the constant disturbance, shown in Figure 4a. Unlike the square wave distrubance rejection, where all of the inputs additionally reach zero, Figure 4b shows that both the constant and gaussian disturbances maintain some non-zero angular position at the end of the simulation time due to the ongoing disturbance.

IV. Discussion

Results from our simulations show that MPC of quadcopter position can effectively reject wind disturbances.

In this work specifically, the cost matrices (P, Q, R) and horizon time (N) chosen result in a system with desirable response characteristics. All disturbance conditions are rejected at steady state. 10-90% rise times of x, y, and z position are less than 1 second. Overshoot is either zero or close to zero (within 10 cm).

More importantly, by including MPC in this system architecture, system behavior can be "tuned" by manipulating the cost matrices and horizon time of the CFTOC. Changing cost matrices will affect the objective of the CFTOC, and changing its horizon time will trade system performance for computational power.

V. Conclusion

In summary, this paper demonstrates the robustness of model-predictive control of quadcopter position and velocity

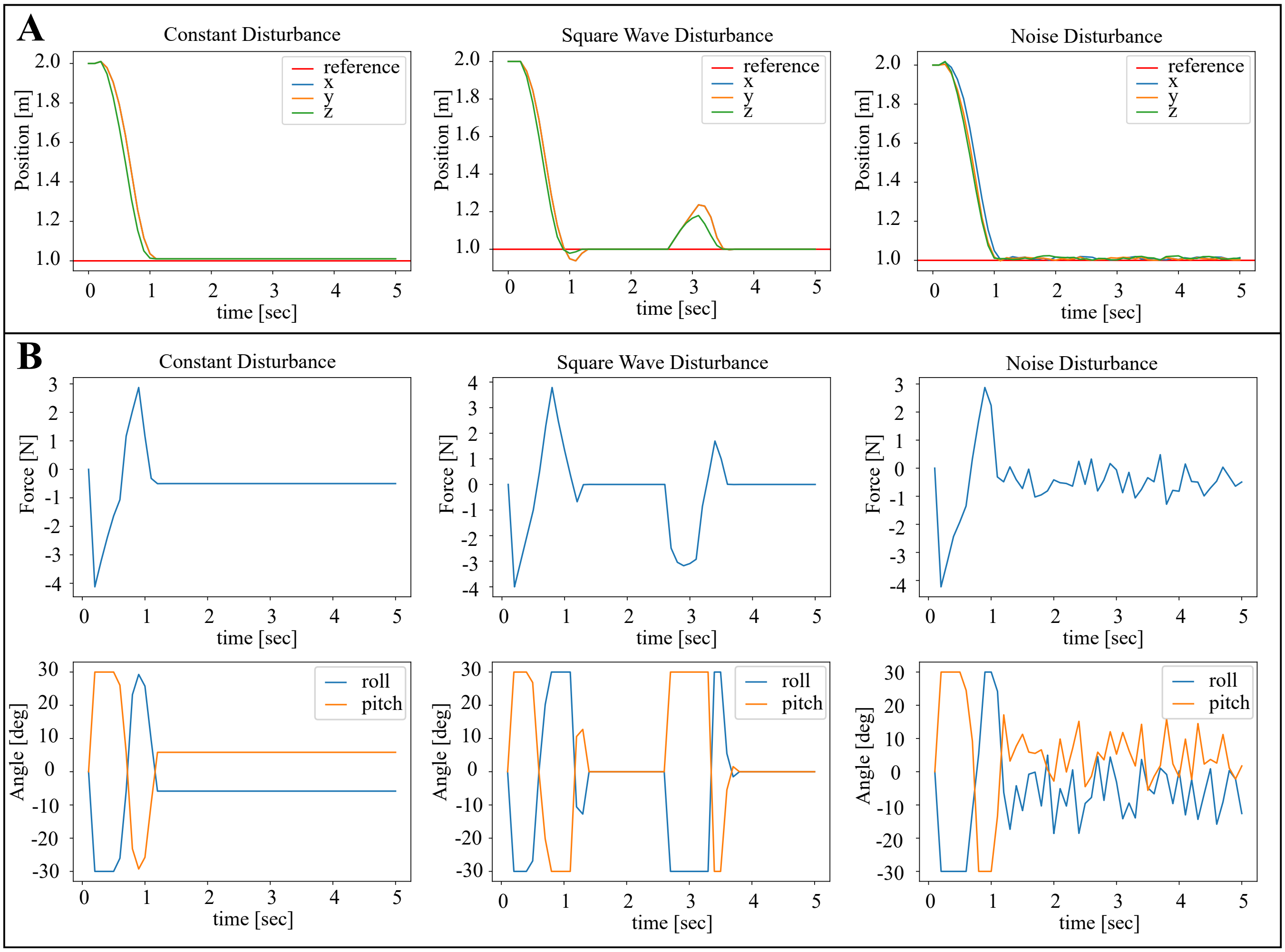

Fig. 4. **Results from Disturbance Rejection** Section **A** presents the position of the quadcopter center of mass for each of the three disturbance cases, including external disturbances. Section **B** presents the optimal inputs as calculated by the MPC (both thrust force and attitude angles) for the three disturbance cases.

when presented with disturbances. This is meant as a first step in addressing closed-loop performance of this class of controller with potential irregularities found in drone flight. In all three cases the system remained stable and was able to converge to a region where there existed a stabilizing Linear Time Invariant (LTI) controller. This therefore verified the closed-loop stability of the MPC. The next step would be to use the same MPC controller to reach an invariant set of a controller able to reject these disturbances. This would work to reduce the steady state error. The effect on the size of the positively invariant set for this autonomous system is left to be investigated. One possible example of a controller could be implementing LQI (Linear-Quadratic-Integrator) control, and seeing how this set can change to see if the control horizon could be shortened.

With the addition of low-level control and tuned LQR/LQI gains for a physical system, the impact of this work could be expanded by testing the robustness of this MPC in natural settings. Lastly, comparison of this MPC's performance in both simulated and physical environments to other control techniques would further our understanding of control techniques for quadcopters in general.

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